A CHESSE PECE

'A Chess Piece'

8"



Patrick Keefe

PREFACE

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 $\begin{tabular}{ll} Figure 1 & The endgame of the computer-versus-computer chess game played by Apple's Chess Application Version 3.15 \\ \end{tabular}$

Introduction

This piece is conceived as a response to Professor Robert Saxton's seminars on structure and proportion in composition, held in the Faculty of Music from October to December 2017. The principles that govern the production and manipulation of the musical material are derived from two main sources: first, a game of chess played by two computers, and second, English polyphony of the late fifteenth and early sixteenth centuries. The title is a reference to both of these sources, spelling 'A Chess Piece' in Middle English. I decided to combine these sources after studying Renaissance musica speculativa works such as Robert Fayrfax's Missa O Quam Glorifica, which reminded me of the logical procedure and inherent mathematical structure of a game of chess.

The Chess Game

The connection between the process of composition and playing a game of chess is not a novel idea. François-André Danican Philidor, for example, was not only a celebrated eighteenth-century composer but also a renowned chess-player in England and France.¹ Sir Walter Parratt is thought to have been a talented player, alongside Sergei Prokofiev, who writes about it frequently in his diaries and himself defeated the noted chess champion José Capablanca.² Using a chess game to inform the compositional process is also not a new notion, a famous example being *Reunion*, the performance 'event' by John Cage, Marcel Duchamp, Alexina Duchamp, David Behrman, Gordon Mumma, David Tudor and Lowell Cross. This used an electronic chessboard to determine 'form and acoustical ambiance'.³ I was aware of this collaboration when I began work on this piece, but wanted to devise a system that could be used to produce music that was not electroacoustic.

The inspiration for my method came from a single figure in David Lewin's *Generalized Musical Intervals and Translations* (2007) and previous work using magic square permutation technique. Lewin's graphic visualises modular harmonic space as a game board. He allocated different spaces on his 8x4 square game board to different pitch classes, which are organized by fifths horizontally and major thirds vertically.⁴ This graphic made me think of a chessboard and how suitable a medium it could be for writing diatonic music, given it is comprised of 8x8 squares.

Previous study of magic square permutation technique provided the next step. I used Peter Maxwell Davies' A Mirror of Whitening Light as a model, repeatedly altering the 8-note set to fill the 8x8 grid. Instead of transposing the intervals to form a transposition matrix, I simply rotated them, starting on the subsequent note of the scale each time. As shown in Figure 2, the scale I used was F major. This choice was informed by the other source material used in the construction of this piece, pre-Reformation English polyphony, as F major is one of the main keys in which that repertory is notated in modern editions.

¹ Reuben Fine – 'Chess and Music' in *Notes* Vol.1 (1944) pg. 41

² The Musical Times Vol. 42 (1901) pg. 731; Orlando Figes — review of Sergey Prokofiev Diaries 1907-1914: Prodigious Youth trans. Anthony Phillips pgs. 9-10

³ Lowell Cross - 'Reunion: John Cage, Marcel Duchamp, Electronic Music and Chess' in Leonardo Music Journal Vol.9, Power and Responsibility: Politics, Identity and Technology in Music (1999) pg. 35

⁴ David Lewin – Generalized Musical Intervals and Translations pg.21

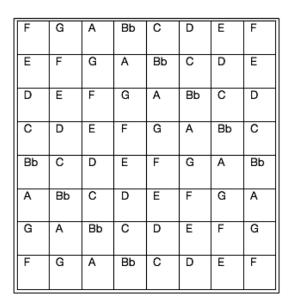


Figure 2 The 8x8 square that maps onto a chessboard, showing an F major scale undergoing cyclical permutation

a8	b8	с8	d8	e8	f8	g8	h8
a7	b7	с7	d7	e7	f7	g7	h7
a6	b6	c6	d6	e6	f6	g6	h6
a5	b5	c5	d5	e5	f5	g5	h5
a4	b4	с4	d4	e4	f4	g4	h4
a3	b3	сЗ	d3	e3	f3	g3	h3
a2	b2	c2	d2	e2	f2	g2	h2
a1	b1	c1	d1	e1	f1	g1	h1

Figure 3 A chessboard using conventional chess labelling for each square

My method maps Figure 2 onto Figure 3, such that a2 represents a G, for example. Moving from one square on the chessboard to another therefore outlines an interval. A knight moving from e2 to f4, for example, corresponds to the interval D-G.

To produce music these pitches need to be combined with note values. In order to acquire these I used a variation on Milton Babbitt's time-point system, which was discussed in Professor Saxton's first seminar (October 2017). Instead of using pitch set numbers to determine points of initiation, I instead make the duration of the interval correspond to the number of chromatic units separating the constituent pitch classes (Figure 4).⁵ In the case of D-G, G is 5 chromatic units from D, and as such the duration of the interval is 5

⁵ Richard Taruskin - *The Oxford History of Western Music: Music in the Late Twentieth Century,* pgs.166-167

rhythmic units. In order to fill 8 minutes, I made the rhythmic unit a quaver. Each pitch class in an interval takes half of the total value. D-G is a 5-quaver-long interval, and therefore G and D would last 2.5 quavers each. In order to incorporate more information from the chess game into this system, I decided that an additional modifier should be applied based on the value of the chess piece being moved. According to Emanuel Lasker (1934), a pawn is worth 1, a knight and bishop both 3, a rook 5, a queen 9 and a king 4.6 I multiplied the number of chromatic units in an interval by the value of the piece tracing it to produce the total duration of the interval. If D-G was the interval traced by a pawn (1) it would still equal a total interval duration of 5 quavers, with D and G lasting 2.5 quavers each. However, if the interval was traced by knight (3) the total interval duration would be 3x5 = 15 quavers, with each note lasting 7.5 quavers.



Figure 4 Babbitt-inspired system for converting pitch numbers into note values. In Babbitt's system a pitch number, say 5 (Bb), would mean the Bb would start on the 5th quaver of the measure (if the above was, say, a bar of 3/2) and last until the initiation of the next pitch at its appropriate time-point. In my system the distance in quavers between two pitches is the note value for that interval. D-G, for example, consists of 5 quavers, reading 'around-the-clock' from D through Eb, E, F, and Gb to G

This system made it possible to turn a game of chess into music by transcribing each move as an interval, forming a continuous line of material until the final move. In a game of chess there are two players, and as such two continuous lines are produced by any one game, one corresponding to White's moves and the other to Black's. Because Apple Inc.'s Chess application provides a comprehensive log of all moves, I used this software to generate the game this piece is based on. Figure 5 shows the complete log of that chess game, along with the interval each move maps to and the resultant total note-value in quavers for that interval.

The need for continuous lines of music was one factor affecting the instrumentation of the work. The other was my other source, early English polyphonic music. I decided that a string quartet would be an appropriate ensemble to realise the chess game, as string instruments can play continuously and are the modern-day equivalent of a consort of viols, the family of instruments being developed in Italy towards the end of the fifteenth century and which were in England by 1510-11.7 I scored the second choir for trombone ensemble for the same aesthetic purpose, with the aim that they would approximate the sound of Renaissance sackbuts.

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⁶ Emanuel Lasker – Lasker's Chess Primer pg. 73

⁷ Ian Woodfield – *The Early History of the Viol* pg. 19; Michael Fleming, John Bryan – *Early English Viols: Instruments, Makers and Music* pg. 60

Black Cantus (Viola)

Move	Interval	Rhythmic Unit (Quavers)
e7-e6	Bb-A	11
d7-d5	A-F	8
h7-h6	E-D	10
Ng8-f6	E-Bb	3(6)
Nf6-d7	Bb-A	3(11)
c7-c5	G-E	9
Nb8-c6	G-F	3(10)
c5xd4	E]-E	12
Nd7-b6	A-E	3(7)
Bf8-e7	D-Bb	3(8)
f7-f6	C-Bb	10
0-0 = (Ke8-g8)+(Rh8-f8)	C-E + F-D	4(4) + 5(9)
f6-f5	Bb-A	11
Bc8-d7	A-A	3(11)
Kg8-h8	E-F	4(1)
Qd8-c8	Bb-A	9(11)
Bd7-e8	A-C	3(3)
a7-a5	E-C	8
Nc6-a7	F-E	3(11)
Rf8-f7	D-C	5(10)
Be7xc5	Bb-E	3(6)
Rf7-c7	c-g]	5(7)
Rc7xc5	G-E	5(9)
Nb6-d7	E-A	3(5)
Be8-f7	C-C	3(12)
Qc8-e8	A-C	9(3)
b7xc6	F-F	12
Nd7-b8	D-C	3(10)
Ra8xa7	Bb-A	5(11)
Qe8-c8	F-D	9(9)
Ra7-a8	A-Bb	5(1)
Bf7-g8	F-A	3(4)
Nb8-d7	C-D	3(2)
Qc8-f8	D-G	9(5)
Ra8-c8	Bb-D	5(4)
Rc8-e8	D-F	5(3)
Bg8-f7	A-F	3(8)
Bf7xe8	F]-F	3(12)
Qf8-c5	G-A	9(2)
Qc5-c1	A-D	9(5)
Qc1-c5	D-A	9(7)
Qc5-a3	A-D	9(5)
g7-g5	G-Eb	8
Qa3-g3	D-C	9(10)
d5-d4	Bb-A	11
	בת-עו	

White Cantus (Violin II)

Move	Interval	Rhythmic Unit (Quavers)
e2-e4	D-F	3
d2-d4	C-E	4
Nb1-d2	G-C	3(5)
e4-e5	F-G	2
Bf1-d3	D-D	3(12)
c2-c3	Bb-C	2
Ng1-e2	E-D	3(10)
c3xd4	C-E	4
0-0 = (Ke1-g1) + (Rh1-f1)	C-E + F-D	4(4) + 5(9)
a2-a3	G-A	2
f2-f4	E-G	3
Qd1-c2	Bb-Bb	9(12)
b2-b4	A-C	3
Nd2-b3	C-Bb	3(10)
Nb3-c5	Bb-E	3(6)
Bc1-b2	A-A	3(12)
Ra1-c1	F-A	5(4)
b4-b5	C-D	2
a3-a4	A-Bb	1
Qc2-b3	Bb-Bb	9(12)
Rc1xc5	A-E	5(7)
Rf1-c1	D-A	5(7)
d4xc5	E]-E	12
Bb2-d4	A-E	3(7)
Qb3-c3	Bb-C	9(2)
c5-c6	E-F	1
b5xc6	D-F	3
Bd4xa7	E-E	3(12)
c6-c7	F-G	2
Qc3-c5	C-E	9(4)
Qc5-e7	A-Eb	9(6)
Ne2-g3	G-C	3(5)
Ng3-h5	C-F	3(5)
Qe7xd7	Eb-D	9(11)
Bd3-a6	G-G	3(12)
c7-c8	C-D	2
Qc8xe8	D-F	9(3)
Rc1-c8	D-D]	5(12)
Kg1-f1	A-G	4(10)
Kf1-f2	G-A	4(2)
Kf2-g3	A-C	4(3)
Kg3-h4	C-Eb	4(3)
Kh4xg5	Eb-Eb	4(12)
Kg5-h4	Eb-Eb	4(12)
Kh4xg3	Eb-C	4(9)
Rc8xe8	D-F]	5(3)
	D-1/1	

Figure 5 The complete log of the computer-versus-computer chess game generated using Apple's Chess application, with the resultant interval and duration in quavers. The essential interval duration is shown in brackets with the multiplying modifier outside them. A 5(4) duration, for example, refers to 5x4=20 quavers

I placed the line traced by White's moves in the second violin and the line traced by Black's moves in the viola. This means that the string quartet has a constant 'inner counterpoint' running through it, produced by the 'counterpoint' of two AI players playing against one another. I call each of these lines a 'cantus' as the melodic material in the outer parts is dependent on them, as was the case in historical polyphonic composition. The 'cantus' lines can only last as long as there were chess moves in the game, and so towards the end of the piece each stop somewhat abruptly, whilst the first violin and cello continue. The chess game is only realised in the inner strings; the rest of the piece, including the brass material, is constructed out of each 'cantus' using a combination of ratios.

The Ratios

Late-Medieval/Early-Renaissance English polyphony seemed an appropriate source to turn to in response to Professor Saxton's seminars, as the music 'shows a profound interest in order, structure and harmony'.8 One of the works studied in the seminars was John Dunstaple's Veni Sancte Spiritus, which contains two taleae to each color, and reduces in the ratio 3:2:1.9 I avoided the use of isorhythm, but wanted to maintain the 3:2:1 structural ratio. Structures in the music of this repertory are often delineated by numbers of measures, and I adopted the same approach in this piece. It is comprised of 168 bars, which divide into 3 sections of 84, 56 and 28 bars respectively. This number was selected as it is divisible by 6 and thus could be separated into a 3:2:1 ratio, and also because it combined with some tempo considerations (that will be discussed shortly) to make the piece last a mathematically exact 8 minutes. The same ratio was used to create the essential melodic material for each section of the work. The number of notes used in each 'head motive' decreases proportionally, as shown in figure 6. Each section uses its respective essential material as a basis for its melodic content, with the last section based on just one interval – a sixth. The three motives can be seen transposed and in juxtaposition in the cello part at the end of the piece, to give the ending a sense of conclusion. The limitation on material in the final section, in having only a single interval as a melodic basis, reflects the limited possible moves towards the end of the chess game as checkmate is approached.



Figure 6 The essential 'head motives', or main melodic elements, of each of the three main sections of the piece. The first section consists of 3x2 notes, the second 2x2 notes, and the last 1x2 notes. The melodic breakdown thus follows the same 3:2:1 ratio that governs the structure of the piece

Roger Bray notes the importance of Boethian theory in late-medieval English polyphony, especially Pythagorean relations such as 1:2 and 3:2.10 The 3:2:1 structural ratio incorporates

⁸ Roger Bray – Music in Britain: the Sixteenth Century pg. 46

⁹ Margaret Bent – *Dunstaple* pg. 53

¹⁰ Roger Bray - Music in Britain: the Sixteenth Century pg. 49

both of these relations. Knowing that 3:2 is a perfect fifth according to Pythagorean tuning, and 2:1 an octave, I decided to use the 3:2:1 ratio to govern modulation as well as structure. I adjust the tonal centre by a difference of a perfect 5th at the point in the structure where the number of measures in a section decreases in the ratio 3:2. The trombones transpose a 4th up, rather than a 5th down, for reasons of orchestration. The 'cantus' parts must change notes at this point to facilitate the transposition – this does not compromise the principle of the chess game however. The *mapping* of the chessboard changes from an F base to a Bb base, but the game board itself does not alter. As an example, the space b7 changes from an F to a Bb, but is still the space b7. At bars 84-85 an F-F interval is suggested in the viola as a pawn moved from b7 to c6. However, the interval is instead F/Bb-Bb, as the transposition occurs part way through the first note. This is still articulating the move from b7 to c6, but the pitch which those spaces map to changes part way through the first note.

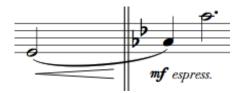


Figure 7 The moment of transposition in the viola 'cantus'. The spaces b7 and c7 both correspond to an F in the F major mapping, and a Bb in the Bb mapping. The piece switches from an F major square to a Bb square on the barline between bars 84 and 85, which happens to be during the first note in the viola's interval at this point. Thus b7 stops meaning F and starts meaning Bb, and consequently the octave leap required by the move to c6 maps up to another Bb

There is no modulation between the final two sections because they are in the ratio 2:1. This represents motion by an octave, and as such there is no change in tonality. The strings do not start suddenly playing at the bottom of their ranges at this point (i.e. generally an octave lower than the previous section), as the 3:2:1 ratio governs modulation, and not general pitch position.

3:2:1 is not the only ratio used to construct this piece, however. Bray notes that 'one of the most important characteristics... [of] the early Tudor mass [is] its capacity for different analyses depending on the combination of neighbouring passages in different ways'." The *Gloria* from Lionel Power's mass *Alma Redemptoris Mater*, from a slightly earlier period, exemplifies how this can work. The first section is comprised of 16 measures followed by 4 measures of rest, then another 16 sung measures followed by 8 of rest, then 14 sung measures. The second section is comprised of 20 measures of sung material followed by 8 of rest, then 38 sung and 8 of rest, then another 12 sung. This could be seen as one section of 58 measures and another of 86, or it could be conceived as a palindromic structure of 20, 38, 28, 38 and 20 measures. I use this principle in my first section by subdividing the '3' of the 3:2:1 ratio into its own 1:2 proportion. The whole structure could therefore be interpreted as a 1:2:2:1 palindrome, which fits not only with the concern of the English polyphonists to achieve symmetries, but also with the inherent symmetry in chess, e.g. each player's layout being the mirror of the other. The structure therefore subdivides from the 84:56:28 structure into a 28:56:56:28 layout.

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¹¹ Roger Bray - Music in Britain: the Sixteenth Century pg. 48

¹² Roger Bray - *Music in Britain: the Sixteenth Century* pg. 47

Just as I used the 3:2:1 ratio to govern modulation and the construction of the 'head motives', I use the 1:2:2:1 ratio to govern more than structure alone. Perhaps the clearest manifestation of the 1:2:2:1 relation in this piece is the introduction of the brass at bar 29 and their removal at bar 141. The number of instrumental choirs thus follows the ratio, starting with 1 in the first subsection, then 2 in the second, then 2 in the third and finally 1 again. The tempi also follow the ratio, starting with 56bpm then progressing to 112bpm in the middle sections before returning to 56bpm at the end. The 56bpm at the start was chosen for two reasons. First, it numerically matches the 56 bars of the middle sections, akin to the numerical similarities often seen in the music of Walter Frye. Second, it combines with the 168 bar structure and its subdivisons to make the piece last a mathematically exact 8 minutes. There are two sections of 28 bars of 4/4 at 56bpm, which corresponds to a duration of 4 minutes, and two sections of 56 bars at 112bpm (twice the material at twice the speed) also equalling 4 minutes. This equates to a total duration of 8 minutes.

The 1:2:2:1 ratio also affects the material the brass play. When they enter they play the content that was just played by the strings, though in double note values. The presentation of the material is therefore in a 1:2 relation in terms of note values. However, as the tempo has also doubled, the material is in effect a canon, with the strings playing new material simultaneously. Halfway through the piece, at the 2:2 part of the 1:2:2:1 ratio, the material not only modulates, but is presented in retrograde to fit with the palindromic nature of the ratio. The brass play the material they have just played a 4th higher and in reverse, forming a retrograde canon, whilst the strings play more new material. Whilst the canons are theoretically exact, I have altered the brass parts by placing the material in different octaves to fit the instrumentation, and adding rests so that the brass players may breathe and rest their lips. Where possible, these rests were added where there was another part playing the same pitch, or in the middle of extended tied passages, to aurally preserve as much of the canon as possible.

The 'free' parts in this composition are the first violin and cello, as they are not bound by the intervals/durations governed by the chess game or by the requirements of canon technique. They are not completely free, however, as they can only play notes belonging to a certain set. This set is obtained from a fraction of one of the two 'cantus' parts, and that fraction changes with each section according to the 1:2:2:1 ratio. The first violin can only use notes from the second violin 'cantus', and the cello can only use notes from the viola 'cantus'. As an example, the first section lasts 28 bars, which is one-sixth of the total length of the piece. The first set is therefore formed from the first sixth of the notes of the cantus. In Figure 5 the end of each fraction of the 'cantus' is marked with a . The set from which material for the first violin can be constructed in the first section is [F, G, Bb, C, D, E] as those are the pitches played by the second violin within the first sixth of its 'cantus'. The lack of an A in this set explains why the melody is split in the opening between the first violin and cello; the violin can't play the last note in the head motive, but the cello is able to as it has an A in its set. For the next section (bars 29 to 84) the proportion of the structure is a third, and therefore the fraction of notes from the cantus is greater, with the next third of the pitches possible for use.

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¹³ Roger Bray - *Music in Britain: the Sixteenth Century* pg. 48

As can be seen in Figure 5, the third portion of the 'cantus', which governs the pitch class set of the third section, crosses over the point of transposition (marked as a blue line). Again, this is just a change in *mapping*. The pitch class set remains the same, but the pitches which those pitch class numbers correspond to changes. Before the blue line, the pitches used in the third proportion of the viola 'cantus' are [0,2,4,7,e] with o referring to an F. After the blue line the pitches used are [0,2,4,7,9,e] with o referring to a Bb. The pitch class set for the whole section is simply the combined [0,2,4,7,9,e]. As the third section doesn't begin until the transposition, the set from which material for the cello can be constructed is [0,2,4,7,9,e] with o corresponding to a Bb. The third fraction of the violin II 'cantus' contains a full diatonic set, and therefore the first violin can play any of the notes in Bb major in the third section.

Aside from 3:2:1 and 1:2:2:1, the final ratio used to govern proportion in the piece is the 'golden' ratio. As discussed in Professor Saxton's seminars, the climax of each section occurs at the moment of golden proportion. This occurs at measure $\frac{\epsilon}{\varnothing}$, where ϵ is the total number of bars in that section and \varnothing is the golden ratio $(\frac{1+\sqrt{5}}{2})$. As an example, the first section is 84 bars long, and as such $\frac{\epsilon}{\varnothing}$ equals 51.914855055, and thus the climax occurs at bar 52.

Structural ratios and the parameters they affect

DUNSTAPLE RATIO	3		2	1
MEASURES	84		56	28
MOTIVE	A		В	С
MELODIC BREAKDOWN	2x3 notes		2x2 notes	2x1 note
TONALITY	F		Bb	Bb
GOLDEN RATIO MEASURE	52		119	157
SUBDIVIDED RATIO	1	2	2	1
MEASURES	28	56	56	28
STRINGS	A		В	С
BRASS		2 A	- 2 A	
TEMPO	56	112	112	56
'CANTUS' PC SET PROPORTION	1/6	1/3	1/3	1/6

Figure 8 Table displaying the 3:2:1 and 1:2:2:1 ratio and the aspects of the music they govern, showing how the subdivided ratio and its effects fit within the wider 3:2:1 ratio. The table illustrates that all the parameters controlled by the ratios alter in sync; for example, first 1:2 proportion causes the brass to enter and the tempo and brass note values to double all at bar 29, the moment in the structure where the number of measures expands from a previous section of 28 bars to a new section of 56 bars. The brass melodic content is presented in the table as 2A to indicate double note values, and -2A to indicate double note values in retrograde

Other Medieval/Renaissance references

As was mentioned previously, the instrumentation of the work is meant to conjure up a Renaissance 'soundworld'. Although the work is not a pastiche of Late-Medieval/Early-Renaissance music, there are a number of features that have been used to deliberately reflect its style. Other than the use of structural ratios, these include the diminution and augmentation of melodic figures, imitation, suspensions, contrapuntal writing, the F/Bb major scoring and melodic shapes such as the falling phrase outlining a perfect 4th at bar 108 in the first violin. The work is a 'tapestry' of sound, with a sense of perpetuity similar to that present in pieces in the Eton Choirbook. The length of the piece and its 'unending' feel are largely a result of the integral structural ratios, as is the case with pre-Reformation English polyphony.

Technical Considerations

- Long sustained notes in the strings should be freely-bowed throughout.
- The notes are beamed and grouped to the crotchet for ease of comprehension. This was advised by the Czigány Quartet, who found the more complicated rhythms easier to navigate if each of the four crotchets in a bar was easily perceivable. Note values that are not made up of an integer number of crotchets are mostly tied rather than dotted to facilitate this. Dots are used for non-integer numbers of quavers, however, as this was thought to be clearer and, again, make the crotchet easier to perceive.
- The only exceptions to the above beaming rule can be found in the first violin at bars 130 and 134, where the beaming is broken before the slurred quavers to preserve their motivic shape.
- If possible, the strings should be placed together near the audience, with the brass together but some distance back from the strings. This is to allow the strings to be in the foreground, with the brass canon functioning more as a kind of 'pad'. Attempts have been made in the notated dynamics to account for balance issues between the brass and strings, but these may need to be adapted in different acoustics to ensure the strings are in the foreground.

10/12/2017

A Chesse Pece

'A Chess Piece'

for String Quartet and Trombone Choir

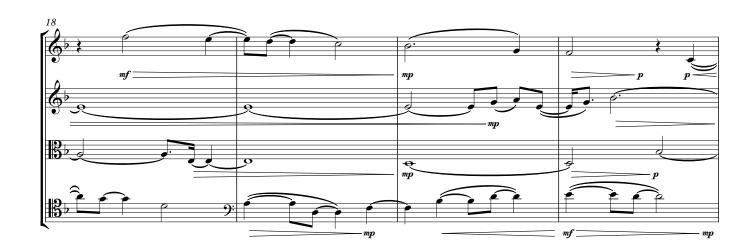
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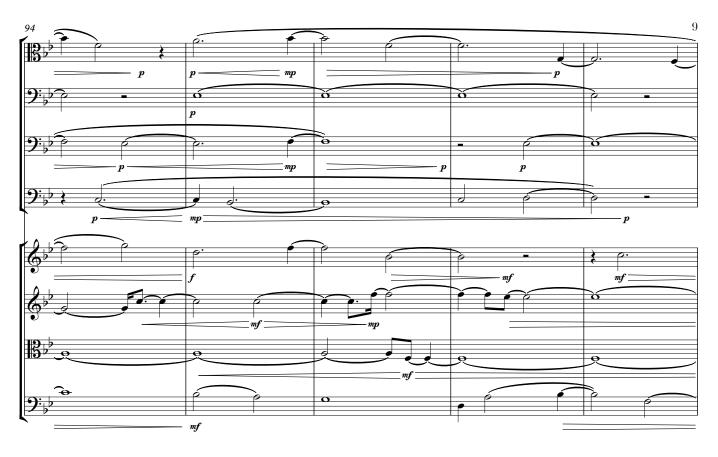




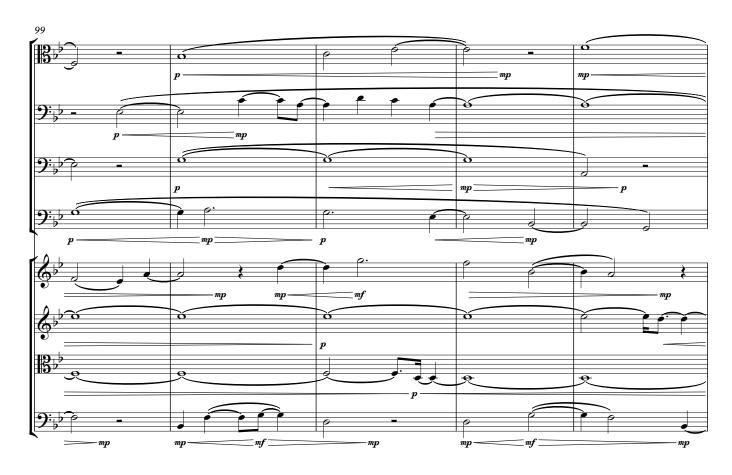






















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